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- 08-01 Identify and Describe Events
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- 08-03 Make Predictions
- 08-04 Find Probabilities
- 08-05 Simulations With Models
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Algebra

09 Algebra Basics

- 09-01 Numbers and Number Operations □ evaluate sum/diff/prod/quot, prime factors
- 09-02 Fractions □ convert to different forms
- 09-03 Variables in Algebra □ write/evaluate expressions
- 09-04 Exponents and Powers □ write/evaluate expressions
- 09-05 Order of Operations □ write/evaluate expressions
- 09-06 Equations and Inequalities □ determine if values are solutions of equations/inequalities
- 09-07 Verbal Models and Algebraic Models □ write equation, translate words → algebraic expression
- 09-08 Sets and Subsets
- 09-09 Union and Intersection of Sets □ Venn diagrams
- 09-10 Problem Solving
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- 10-01 **The Real Number Line** □ comparisons ($<$, $>$, $=$), absolute value
- 10-02 **Addition of Real Numbers**
- 10-03 **Subtraction of Real Numbers**
- 10-04 **Data: Matrix Representation**
- 10-05 **Multiplication of Real Numbers** □ evaluate expressions
- 10-06 **Real Number Properties**
- 10-07 **Division of Real Numbers** □ evaluate expressions
- 10-08 **Compound Fractions**
- 10-09 **Data: Rates and Ratios** □ applications
- 10-10 **Data: Graphical Representation**
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- 11 Solving Linear Equations**
 - 11-01 **Solving Equations**
 - 11-02 **Solving Equations with Variables on Both Sides**
 - 11-03 **Problem Solving**
 - 11-04 **Solving Equations that Involve Decimals**
 - 11-05 **Parametric Equations and Formulae**
 - 11-06 **Data: Scatter Plots** □ some linear best fit
 - 11-99 Associated problems in Chapter 11

- 12 Graphing Linear Equations**
 - 12-01 **The Rectangular Coordinate System** □ graph ordered pairs, quadrant identification
 - 12-02 **Graphs: One Variable** □ horizontal/vertical lines
 - 12-03 **Graphs: Two Variables** □ determine if points are on a line, graph using table of values
 - 12-04 **Intercepts** □ intercepts from graph
 - 12-05 **Slope As Rate of Change** □ slope from graph/points
 - 12-06 **Slope-Intercept Form** □ slopes/intercepts from graphs/data, modeling
 - 12-07 **Solutions and x -Intercepts**
 - 12-08 **Graphs: Absolute Value** □ analysis of graph, intercepts, equation from graph, find values
 - 12-09 **Solving Absolute Value Equations**
 - 12-10 **Applications: Midpoints, Parallel/Perpendicular Lines**
 - 12-11 **Graphs: Three or More Variables**
 - 12-99 Associated problems in Chapter 12

- 13 Writing Linear Equations**
 - 13-01 **Write Equations: Slope-Intercept**
 - 13-02 **Write Equations: Slope and a Point**
 - 13-03 **Write Equations: Two Points**
 - 13-04 **Standard and Two-Intercept Forms**
 - 13-05 **Parallel and Perpendicular Lines**
 - 13-06 **Problem Solving**
 - 13-07 **Data: Fitting a Line to Data**
 - 13-99 Associated problems in Chapter 13

- 14 Linear Inequalities**
 - 14-01 **Inequalities in One Variable** □ solve, graph
 - 14-02 **Problem Solving** □ is/is not solution of inequality
 - 14-03 **Compound Inequalities** □ logical and/or, solve, graph
 - 14-04 **Absolute Value Inequalities** □ solve, write inequality from graph
 - 14-05 **Graphing Linear Inequalities**
 - 14-06 **Data: Time Lines, Picture Graphs, and Circle Graphs**
 - 14-99 Associated problems in Chapter 14

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15 Systems of Equations and Inequalities

- 15-01 **Solve: Graphing**
- 15-02 **Solve: Substitution**
- 15-03 **Solve: Linear Combinations**
- 15-04 **Problem Solving**
- 15-05 **System Analysis** □ number of solutions, consistent/inconsistent, points are/are not solutions
- 15-06 **Systems of Inequalities**
- 15-07 **Data: Linear Programming** □ (22:08 *simplex method*)
- 15-08 **Multivariable Systems** □ dependent equations, curve fitting
- 15-09 **Partial Fractions**
- 15-99 Associated problems in Chapter 15

16 Powers and Exponents

- 16-01 **Multiplication Properties of Exponents** □ $a^m a^n = a^{m+n}$, $(ab)^n = a^n b^n$, $(a^m)^n = a^{mn}$
- 16-02 **Negative and Zero Exponents**
- 16-03 **Division Properties of Exponents** □ $\frac{a^m}{a^n} = a^{m-n}$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- 16-04 **Scientific Notation** □ scientific \leftrightarrow decimal, applications
- 16-05 **Problem Solving: Compound Interest** □ $A = P\left(1 + \frac{r}{n}\right)^{nt}$
- 16-06 **Data: Exponential Growth and Decay**
- 16-99 Associated problems in Chapter 16

17 Quadratic Equations

- 17-01 **Square Roots and the Pythagorean Theorem**
- 17-02 **Quadratic Equations: Square Roots**
- 17-03 **Graphs of Quadratic Equations** □ end behavior
- 17-04 **The Quadratic Formula**
- 17-05 **Equation Analysis: the Discriminant**
- 17-06 **Quadratic Inequalities** □ solve, graph
- 17-07 **Data: Comparing Models**
- 17-08 **Problem Solving**
- 17-99 Associated problems in Chapter 17

18 Polynomials and Factoring

- 18-01 **Polynomials** □ classify
- 18-02 **Adding and Subtracting Polynomials** □ linear combinations
- 18-03 **Multiplying Polynomials**
- 18-04 **Multiplication: Special Cases** □ patterns: squaring, diff squares
- 18-05 **Factoring: Greatest Common Factor** □ (also 08:01)
- 18-06 **Factoring: Difference of Squares**
- 18-07 **Factoring: Quadratic Trinomials**
- 18-08 **Factoring: Sum and Difference of Cubes**
- 18-09 **Factoring: Grouping and Other Methods**
- 18-10 **Solve Equations: Factoring**
- 18-11 **Solve Equations: Completing the Square**
- 18-12 **Imaginary Numbers** □ standard form, complex conjugate
- 18-13 **Solve Polynomial Equations** □ real and imaginary solutions
- 18-14 **Polynomial Inequalities, Systems**
- 18-99 Associated problems in Chapter 18

19 Rational Expressions and Equations

- 19-01 **Ratios and Proportions**
- 19-02 **Percents**

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- 19-03 **Direct and Inverse Variation**
- 19-04 **Data: Probability**
- 19-05 **Simplify Rational Expressions** □ reduce fractions
- 19-06 **Multiply/Divide Rational Expressions** □ factoring
- 19-07 **Add/Subtract Rational Expressions** □ find LCM/LCD
- 19-08 **Compound Fractions** □ mult by clever form of one
- 19-09 **Dividing Polynomials** □ divide by monomial, long division
- 19-10 **Rational Equations** □ value is/is not a solution, solve, zeros
- 19-11 **Rational Inequalities** □ value is/is not a solution, solve
- 19-99 Associated problems in Chapter 19

- 20 Functions and Their Graphs**
- 20-01 **Functions and Relations** □ functional notation, domain/range, *one-to-one*, extrema, evaluate, mapping, vertical line test
- 20-02 **Properties** □ even/odd, increasing/decreasing/constant, max/min, symmetry, intercepts
- 20-03 **Linear Functions** □ find equation, graph, evaluate functional notation
- 20-04 **Exponential Functions** □ increasing/decreasing, graph
- 20-05 **Quadratic Functions** □ evaluate, solve, complete square, extrema
- 20-06 **Rational Functions** □ domain/range, graph
- 20-07 **Function Operations** □ add/subtr/mult/div, evaluate
- 20-08 **Piecewise Defined Functions** □ absolute value
- 20-09 **Composition of Functions**
- 20-10 **Other Functions** □ radical, other polynomial, greatest integer
- 20-11 **Inverses** □ horizontal line test, graphs, equations, domain/range restrictions
- 20-12 **Transformations of Graphs** □ shrink, stretch, shifts, etc.
- 20-13 **Recursive Functions and Finite Differences** □ first/second differences, *factorials*
- 20-14 **Data: Stem-and-Leaf and Box-and-Whisker Plots**
- 20-15 **Data: Central Tendency** □ mean, median, mode
- 20-16 **Data: Models and Scatter Plots**
- 20-17 **Graphical Analysis** □ end behavior, limits, extrema
- 20-18 **Rationalize Numerators, Denominators**
- 20-19 **Difference Quotient As Rate of Change** □ secant line
- 20-99 Associated problems in Chapter 20

- 21 Radicals and Radical Functions** □ (chap 16: *Exponents*)
- 21-01 **Simplifying Radicals** □ $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$
- 21-02 **n^{th} Roots and Rational Exponents** □ $a^{m/n} = (a^{1/n})^m$
- 21-03 **Operations with Radicals** □ add/subtr/mult/divide/powers, (20:18 *Rationalize*)
- 21-04 **Radical Equations** □ transformations, domain/range, value is/is not solution, equation \leftrightarrow graph
- 21-05 **Graphing Square Root and Cube Root Functions**
- 21-06 **Applications: Distance, Pythagorean Theorem**
- 21-07 **Inverse of Radical Functions**
- 21-99 Associated problems in Chapter 21

- 22 Matrices and Determinants**
- 22-01 **Data: Matrix Operations** □ order, add/subtr, transpose, linear combination, equiv matrices, recognize reduced matrices
- 22-02 **Augmented Matrices and Row Operations** □ row echelon form
- 22-03 **Matrix Multiplication**
- 22-04 **Determinants** □ area of triangle
- 22-05 **Matrix Equations** □ write, solve
- 22-06 **Identity and Inverse Matrices** □ find inverses, solve matrix equations
- 22-07 **Solving Systems: Inverse Matrices**

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- 22-08 **Solving Systems: Augmented Matrices** □ Gauss-Jordan (reduced triangular form)
- 22-09 **Solving Systems: Cramer's Rule**
- 22-10 **Linear Programming: Simplex Method**
- 22-11 **Applications** □ area of triangle, collinear points, encode/decode messages, lines in plane, partial fractions
- 22-99 Associated problems in Chapter 22

- 23 Exponential and Logarithmic Functions**
- 23-01 **Exponential Functions** □ (also 52:02)
- 23-02 **Logarithmic Functions** □ (also 52:04), graphs, intercepts, etc.
- 23-03 **Properties of Logarithms** □ $\log m^n$, $\log\left(\frac{m}{n}\right)$, $\log(mn)$
- 23-04 **The Natural Base e**
- 23-05 **Natural Logarithms**
- 23-06 **Inverses** □ $\log_a b = n \leftrightarrow a^n = b$
- 23-07 **Exponential and Logarithmic Equations**
- 23-08 **Data: Logistics Growth/Decay Models** □ $P = \frac{c}{1+a e^{-bt}}$
- 23-09 **Exponential and Logarithmic Models** □ $A = A_0 e^{\pm kt}$, Newton's Law of Cooling: $u(t) = T + (u_0 - T)e^{kt}$, $k < 0$
- 23-10 **Data: Nonlinear Models**
- 23-11 **Interest: Comparisons and Continuous Compounding** □ (16:05 *Cpd Interest Formula*)
- 23-12 **Present/Future Values of an Annuity: Amortization** □ $P = A\left(1 + \frac{r}{n}\right)^{-nt}$, $A = P \frac{(1+i)^n - 1}{i}$, continuous compounding: $P = A e^{-rt}$, monthly installments: $M = P \frac{1}{1 - \left(\frac{1}{1+i}\right)^{12t}}$
- 23-13 **Systems of Exp/Log Equations**
- 23-99 Associated problems in Chapter 23

- 24 Polynomial Functions**
- 24-01 **Operations with Polynomials** □ add/subtr/mult
- 24-02 **Graphs of Polynomial Functions** □ domain/range, end behavior, intercepts, vertices, asymptotes, extrema, symmetry, (20:01 *one-to-one*)
- 24-03 **Synthetic Division** □ missing factors, remainder/factor theorems, evaluate polynomial at a point
- 24-04 **Real Zeros, Factors, and Solutions** □ number/find real zeros/solutions, intermediate value theorem
- 24-05 **Fundamental Theorem of Algebra** □ complex zeros, factors, solutions, conjugate pairs theorem (18:12 *Imaginary Numbers*)
- 24-06 **Mathematical Modeling**
- 24-07 **Data: Measures of Dispersion** □ standard deviation, range
- 24-99 Associated problems in Chapter 24

- 25 Rational Functions** □ (20:06 *Domain*)
- 25-01 **Graphs of Rational Functions** □ zeros, asymptotes, intercepts, extrema, transformations, limits
- 25-02 **Inverse and Joint Variation** □ $y \propto \frac{1}{x}$, $y \propto$ two variables
- 25-03 **Data: Modeling**
- 25-04 **Problem Solving**
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- 26 Quadratic Relations: Conics**
- 26-01 **Parabolas** □ (also 20:05), vertex, focus, directrix
- 26-02 **Circles** □ center, radius
- 26-03 **Ellipses** □ foci, vertices, eccentricity
- 26-04 **Hyperbolas** □ foci, vertices
- 26-05 **Standard Form of General Conics**
- 26-06 **Classifying Conics** □ equation \leftrightarrow graph
- 26-07 **Rotation and Systems of Quadratic Equations** □ intersections

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26-08 **Inequalities and Systems** □ graph inequalities, solve systems

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27 Sequences and Series □ (20:13 factorials)

27-01 **Sequences, Series, and Summations**

27-02 **Partial Sums Formulae**

27-03 **Arithmetic Sequences and Series** □ common difference d , $a_n = a_{n-1} + d$, $S_n = \frac{n(a_1 + a_n)}{2}$

27-04 **Geometric Sequences and Finite Series** □ common ratio r , $a_n = ra_{n-1}$, $S_n = a_1 \frac{1-r^n}{1-r}$

27-05 **Infinite Geometric Series** □ $\sum_{n=0}^{\infty} a_1 r^n = \frac{a_1}{1-r}$, where $|r| < 1$, repeating dec \rightarrow rational number

27-06 **Identifying Series Types** □ first, second differences

27-07 **Modeling**

27-08 **Combinations** □ ${}_n C_r = \frac{n!}{(n-r)!r!}$

27-09 **Mathematical Induction** □ $f(k+1)$ from $f(k)$

27-10 **The Binomial Theorem** □ Pascal's Triangle

27-11 **Data: The Algebra of Finance**

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28 Probability and Statistics

28-01 **Permutations and Regression** □ ${}_n P_r = \frac{n!}{(n-r)!}$

28-02 **Simple Probability: Counting Principles** □ sample space

28-03 **Compound Probability: Unions and Intersections** □ complementary prob: $P(A') = 1 - P(A)$,
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)P(A|B)$,

28-04 **Independent Events** □ $P(A \cap B) = P(A)P(B)$, expected value: $V = \sum_{i=1}^n P_i n_i$

28-05 **Odds and Mathematical Expectation**

28-06 **Data: Central Tendency** □ mean, median, mode

28-07 **Data: Dispersion** □ variance ν , density function σ , quartiles

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Geometry

29 Geometry Basics

29-01 **Points, Lines and Planes**

29-02 **Segments and Rays**

29-03 **Angles**

29-04 **Optical Illusions**

29-05 **Shapes and Patterns**

29-06 **Congruence and Similarity**

29-07 **Symmetry** □ rotational, midpt, lines of symmetry

29-08 **Polygons**

29-09 **Triangles**

29-10 **Special Quadrilaterals**

29-11 **Space Geometry**

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30 Lines and Angles □ *midpoints* (12:10), *slopes* (12:05), *vectors* (47:01)

30-01 **Line Relationships**

30-02 **Angle Relationships** □ vertical angles

30-03 **Segment/Angle Postulates** □ segment/angle addition

30-04 **Segment/Angle Relationships** □ congruence, midpoint, angle bisector, perpendiculars

30-05 **Parallel Properties**

30-06 **Perpendicular Properties**

30-07 **Proving Parallel, Perpendicular**

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31 Construction Techniques

31-01 Duplicating Segments and Angles

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31-04 Angle Bisectors

31-05 Parallel Lines

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32 Triangles

32-01 Special Segments \square bisectors, medians, altitudes, midsegments

32-02 Points of Concurrency \square centroid, circumcenter, incenter, orthocenter

32-03 Midsegment Properties

32-04 Special Triangles \square equilateral, isosceles, scalene, acute, obtuse

32-05 Angle Properties \square sum, exterior angle

32-06 Isosceles Triangles

32-07 Right Triangles

32-08 Inequalities in One Triangle

32-09 Inequalities in Two Triangles \square Hinge Theorem

32-10 *SSS*, *SAS*, and *SSA* Congruence

32-11 *ASA*, *SAA*, and *AAA* Congruence

32-12 Problem Solving

32-13 Triangle Constructions \square points of concurrency

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33 Polygons

33-01 Polygon Classification \square regular, quadrilateral, *etc.*

33-02 Angles of a Polygon

33-03 Midsegments

33-04 Parallelogram Properties

33-05 Prove Quadrilaterals are Parallelograms

33-06 Special Parallelograms \square rhombus, rectangle, square

33-07 Trapezoids

33-08 Kites

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34 Circles

34-01 Defining Circles

34-02 Chords

34-03 Tangents

34-04 Central Angles and Arcs

34-05 Arcs and Chords

34-06 Inscribed Angles

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- 35-03 Symmetry
- 35-04 Reflection
- 35-05 Rotation
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- 35-07 Glide Reflections and Composition
- 35-08 Identity and Inverse Transformations
- 35-09 Applications: Tessellations □ frieze patterns
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- 36 Similarity**
 - 36-01 Ratio and Proportion
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 - 36-03 Similarity
 - 36-04 Similar Polygons
 - 36-05 Similar Triangles □ *AA*
 - 36-06 Proving Similar Triangles □ *SSS, SAS*
 - 36-07 Proportions in Similar Triangles □ corresponding parts
 - 36-08 Proportions: Area and Volume
 - 36-09 Proportional Segments: Parallel Lines
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- 37 Right Triangles**
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 - 37-04 Special Right Triangles
 - 37-05 Multiples of Right Triangles
 - 37-06 Distance in Coordinate Geometry
 - 37-07 Circles and the Pythagorean Theorem
 - 37-08 Problem Solving
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- 38 Planar Measurements**
 - 38-01 Areas and Perimeters of Polygons
 - 38-02 Areas of Parallelograms and Triangles □ Heron's formula:
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
 - 38-03 Areas of Other Quadrilaterals □ trapezoids, kites
 - 38-04 Areas of Regular Polygons
 - 38-05 Circumference and Arc Length of Circles □ $\frac{\ell}{\text{circum}} = \frac{\theta}{360^\circ}$
 - 38-06 Areas of Circles, Sectors, Segments
 - 38-07 Application: Area of Similar Polygons
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- 39 Solid Geometry**
 - 39-01 Polyhedrons, Prisms, and Pyramids
 - 39-02 Solids with Curved Surfaces
 - 39-03 Surface Area: Prisms and Cylinders
 - 39-04 Surface Area: Pyramids and Cones
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- 40-01 **Loci in Planes**
- 40-02 **Loci in Space**
- 40-03 **Problem Solving**
- 40-04 **Mathematical Models**
- 40-05 **Cross Sections** □ circle, *etc.*, in cone
- 40-06 **Applications: Pappus', Simson's Theorems**
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41 Logic

- 41-01 **Valid Reasoning** □ deductive, inductive
- 41-02 **Conditional Statements**
- 41-03 **The Law of Syllogism** □ $p \rightarrow q$ and $q \rightarrow r \Rightarrow p \rightarrow r$
- 41-04 **Styles of Proofs** □ paragraph, flow, two-column
- 41-05 **The Converse, Inverse and Contrapositive**
- 41-06 **Direct Proofs** □ algebraic properties
- 41-07 **Conditional Proofs**
- 41-08 **Indirect Proofs**
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42 Geometric Proof

- 42-01 **Premises and Postulates of Geometry**
- 42-02 **Geometric Proofs**
- 42-03 **Proving Angle Conjectures**
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43 Sequences of Proofs

- 43-01 **From Conjecture to Proof**
- 43-02 **Proving the Triangle Sum Conjecture**
- 43-03 **Proving Circle Conjectures**
- 43-04 **Proving the Pythagorean Theorem**
- 43-05 **Indirect Geometric Proofs**
- 43-06 **Proof with Coordinate Geometry**
- 43-07 **Midsegment Conjectures**
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Trigonometry

44 Trigonometric Functions

- 44-01 **Radian and Degree Measure** □ central angle, standard position, coterminal angles, quadrants, complement/supplement of radian measure
- 44-02 **Using Radian Measure** □ $\omega = \frac{\theta}{t}$, $s = r\theta$, $v = r\omega$, area of sector: $A = \frac{1}{2}r^2\theta$
- 44-03 **Trigonometric Functions: The Unit Circle** □ wrapping function (x, y, r) , exact values for multiples of quadrant angles and special angles
- 44-04 **Properties** □ domain/range, fundamental period, pos/neg in quadrants, even/odd properties, find trig functions using basic identities (reciprocal, quotient, Pythagorean)

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- 44-05 **Using Properties: Exact Values** □ exact values given two functions or one function and sign of another
- 44-06 **Right Triangle Trigonometry** □ opposite, adjacent, hypotenuse, special angles
- 44-07 **Trigonometric Functions of Any Angle** □ reference angle (opp, adj, hyp)
- 44-08 **Graphs of Sine and Cosine Functions** □ $y = A f(\omega t) + B$
- 44-09 **Graphs of Other Trigonometric Functions**
- 44-10 **Amplitude, Period, Phase Shift** □ $y = A f(\omega t + \phi) + B$
- 44-11 **Inverse Trigonometric Functions** □ solve inverse trig equations, evaluate inverses using calculator
- 44-12 **Composition** □ inverse trig \rightarrow algebraic function
- 44-13 **Problem Solving** □ angle of elevation/depression, bearings
- 44-14 **Data: Curve Fitting**
- 44-99 Associated problems in Chapter 44
- 45 Analytic Trigonometry**
- 45-01 **Fundamental Identities** □ simplify expressions using reciprocal, quotient, Pythagorean, even/odd identities
- 45-02 **Trig Substitution**
- 45-03 **Verifying Trigonometric Identities**
- 45-04 **Equations: One Trig Function**
- 45-05 **Quadratic, Other Linear Trig Equations**
- 45-06 **Sum and Difference Formulae** □ $f(\alpha \pm \beta)$, complementary properties: $f(\frac{\pi}{2} - \theta)$
- 45-07 **Double- and Half-Angle Formulae** □ $f(2\theta)$, $f(\frac{\theta}{2})$,
- 45-08 **Product-Sum Formulae** □ $\sin \alpha \sin \beta$, $\cos \alpha \cos \beta$, $\sin \alpha \cos \beta$, $\sin \alpha \pm \sin \beta$, $\cos \alpha \pm \cos \beta$
- 45-09 **Graphs and Limits** □ difference quotient
- 45-10 **Systems**
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- 46 Additional Topics in Trigonometry**
- 46-01 **Law of Sines** □ $\frac{\sin A}{a} = \frac{\sin B}{b}$, *SAA*, *ASA*, *SSA* triangles
- 46-02 **Law of Cosines** □ $c^2 = a^2 + b^2 - 2ab \cos C$, *SAS*, *SSS* triangles
- 46-03 **Area of Triangle** □ (38:02 *Heron's formula*)
- 46-04 **Simple Harmonic and Damped Motion** □ combining waves
- 46-05 **Polar Coordinates** □ (r, θ) , rectangular/polar conversion: $x = r \cos \theta$, $y = r \sin \theta$
- 46-06 **Polar Equations, Graphs** □ symmetry, cardioids, limaçons, rose curves, lemniscates, spirals
- 46-07 **Polar Equations of Conics**
- 46-08 **Parametric Equations** □ $x = f(t)$, $y = g(t)$, cycloid
- 46-99 Associated problems in Chapter 46
- 47 Vectors**
- 47-01 **Vector Components** □ magnitude, direction angle, unit vector, algebraic: $\vec{v} = \langle a, b \rangle$, position: $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$, equality, $\vec{v} = v_x \hat{i} + v_y \hat{j}$
- 47-02 **Vector Algebra** □ add/subtr, scalar multiple, commutative, associative, additive identity/inverse properties
- 47-03 **Problem Solving** □ bearings
- 47-04 **Complex Plane: Trig Form** □ (also 18:12, 24:05) $z = r(\cos \theta + i \sin \theta)$, modulus, angle
- 47-05 **Complex Plane: Products, Quotients** □ products: $z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$, quotients: $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$
- 47-06 **Complex Plane: Roots** □ n distinct complex roots of z : $z_k = \sqrt[n]{r} [\cos(\frac{\theta_0}{n} + \frac{2k\pi}{n}) + i \sin(\frac{\theta_0}{n} + \frac{2k\pi}{n})]$
- 47-07 **Complex Plane: DeMoivre's Theorem** □ unit vectors, direction/magnitude, $z^n = r^n (\cos \theta + i \sin \theta)$
- 47-08 **Vectors in Space** □ distance, position vectors, algebraic manipulations, direction angle, unit vectors, $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

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- 47-09 **Dot Product** □ angle between vectors, parallel, orthogonal, decompose into two orthogonal vectors, algebraic properties, direction cosines
- 47-10 **Cross Product** □ determinants, find vector orthogonal to two vectors, area of parallelogram, algebraic/geometric properties
- 47-99 Associated problems in Chapter 47

Differentiation

48 Limits and Their Properties

- 48-01 **Estimating Limits**
- 48-02 **Finding Limits Graphically and Numerically**
- 48-03 **Evaluating Limits Analytically** □ ϵ - δ definition
- 48-04 **Continuity and One-Sided Limits** □ continuity properties, Intermediate Value Theorem
- 48-05 **Infinite Limits** □ vertical asymptotes
- 48-06 **Tangents, Velocities and Other Rates of Change**
- 48-99 Associated problems in Chapter 48

49 Differentiation

- 49-01 **The Derivative and the Tangent Line Problem** □ derivative by definition
- 49-02 **Basic Differentiation Rules and Rates of Change**
- 49-03 **Product and Quotient Rules, Higher-Order Derivatives**
- 49-04 **Derivative of Trig Functions**
- 49-05 **The Chain Rule** □ general power rule
- 49-06 **Implicit Differentiation**
- 49-07 **Related Rates** □ problem solving; average, instantaneous rates of change
- 49-99 Associated problems in Chapter 49

50 Applications of Differentiation

- 50-01 **Extrema on an Interval** □ relative extrema, critical numbers
- 50-02 **Rolle's Theorem and the Mean Value Theorem**
- 50-03 **Increasing and Decreasing Functions** □ first derivative test
- 50-04 **Concavity and the Second Derivative Test** □ inflection points
- 50-05 **Limits at Infinity** □ horizontal asymptotes
- 50-06 **A Summary of Curve Sketching**
- 50-07 **Optimization Problems** □ applied max/min problems
- 50-08 **Newton's Method** □ algebraic solutions of polynomial equations
- 50-09 **Differentials** □ linear approximations, error propagation
- 50-10 **Business and Economics Applications** □ marginals
- 50-99 Associated problems in Chapter 50

Integration

51 Integration

- 51-01 **Antiderivatives** □ initial conditions, particular solutions
- 51-02 **Area and Distance** □ Sigma notation, area of plane region, upper, lower sums
- 51-03 **Riemann Sums and Definite Integrals** □ properties, $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*)\Delta x$
- 51-04 **The Fundamental Theorem of Calculus** □ MVT for integrals, average value of a function
- 51-01 **Indefinite Integrals, Net Change Theorem** □ basic rules, $\int_b^a F'(x)dx = F(b) - F(a)$
- 51-05 **Integration by Substitution** □ change of variables, general power rule, even/odd functions
- 51-06 **Numerical Integration** □ trapezoidal rule, Simpson's rule, error analysis
- 51-99 Associated problems in Chapter 51

52 Logarithmic, Exponential, Transcendental

- 52-01 **Inverse Functions** □ (see 20:11) existence, derivative

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- 52-02 **Exponential Functions** □ (see chap 23) def of e
- 52-03 **Logarithmic Functions** □ (see chap 23)
- 52-04 **Differentiation of Exponential Functions**
- 52-05 **Differentiation of Logarithmic Functions**
- 52-06 **Integration and Exponential Functions**
- 52-07 **Integration and Logarithmic Functions**
- 52-08 **Differential Equations** □ growth and decay, logistics
- 52-09 **Inverse Trig Functions and Differentiation** □ (see 44:11)
- 52-10 **Inverse Trig Functions and Integration**
- 52-11 **Hyperbolic Functions** □ inverse hyperbolic functions
- 52-99 Associated problems in Chapter 52

53 Applications of Integration

- 53-01 **Area of a Region Between Two Curves**
- 53-02 **Volume: The Disc Method** □ washers, known cross sections
- 53-03 **Volume: The Shells Method**
- 53-04 **Arc Length and Surfaces of Revolution**
- 53-05 **Work** □ constant, variable forces
- 53-06 **Moments, Centers of Mass, and Centroids** □ 1D, 2D, planar lamina; Theorem of Pappus
- 53-07 **Fluid Pressure and Fluid Force**
- 53-08 **Average Value of a Function** □ average rate of change
- 53-09 **Applications to Economics and Biology** □ future, present value
- 53-10 **Probability** □ probability density, average value, normal distribution
- 53-99 Associated problems in Chapter 53

54 Integration Techniques, Improper Integrals

- 54-01 **Integration by Parts** □ tabular method
- 54-02 **Trigonometric Integrals** □ powers of trig functions, sine-cosine products w/ different angles
- 54-03 **Trigonometric Substitution** □ applications
- 54-04 **Partial Fractions** □ linear, quadratic factors
- 54-05 **Tables and Other Integration Techniques** □ reduction formulae, rational functions of sine/cosine
- 54-06 **Indeterminate Forms and L'Hospital's Rule**
- 54-07 **Improper Integrals** □ infinite limits, discontinuities
- 54-99 Associated problems in Chapter 54

55 First-Order Differential Equations

- 55-01 **Modeling DEs** □ confirm solutions for DEs
- 55-02 **Direction Fields and Euler's Method**
- 55-03 **Separable Equations** □ $\frac{dy}{dx} = g(x)f(y)$
- 55-04 **Population Growth Models** □ $\frac{dP}{dt} = kP$, logistics: $\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$
- 55-05 **Linear DEs** □ $y' = P(x)y + Q(x)$, integrating factor $I(x) = e^{\int P(x)dx}$, electric circuits
- 55-06 **Predator-Prey Systems**
- 55-99 Associated problems in Chapter 55

56 Infinite Series

- 56-01 **Sequences** □ limits, pattern recognition, monotonic and bounded sequences
- 56-02 **Series and Convergence** □ infinite, geometric series; n^{th} term divergence test
- 56-03 **The Integral Test and p-Series** □ harmonic series
- 56-04 **Comparisons of Series** □ direct, limit comparison tests
- 56-05 **Alternating Series** □ remainder; absolute, conditional convergence; rearrangement
- 56-06 **The Ratio and Root Tests** □ testing strategies
- 56-07 **Taylor Polynomials and Approximations** □ Maclaurin polynomial, Taylor remainder
- 56-08 **Power Series** □ radius, interval of convergence; endpoint convergence; differentiation and integration

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- 56-09 **Representation of Functions by Power Series** [] geometric power series, operations
- 56-10 **Taylor and Maclaurin Series** [] derive Taylor from basic list
- 56-11 **The Binomial Series**
- 56-99 Associated problems in Chapter 56

- 57 Plane and Polar Curves, Parametric Equations**
 - 57-01 **Plane Curves and Parametric Equations**
 - 57-02 **Parametric Equations and Calculus** [] slope and tangent lines, arc length, area of surface of revolution
 - 57-03 **Polar Coordinates and Polar Graphs** [] (see 46:05) slope and tangent lines
 - 57-04 **Area and Arc Length in Polar Coordinates** [] points of intersection of polar graphs, area of surface of revolution
 - 57-05 **Polar Equations of Conics and Kepler's Laws** [] (see chap 23, 46)
 - 57-99 Associated problems in Chapter 57

- 58 Vectors and the Geometry of Space**
 - 58-01 **Vectors in the Plane** [] components, operations, standard unit vectors, applications
 - 58-02 **Space Coordinates and Vectors in Space** [] applications
 - 58-03 **The Dot Product of Two Vectors** [] angle between vectors, direction cosines, projections and vector components, work
 - 58-04 **The Cross Product of Two Vectors in Space** [] triple scalar product
 - 58-05 **Lines and Planes in Space** [] sketching, distances between points, planes, and lines
 - 58-06 **Surfaces in Space** [] cylindrical and quadric surfaces, surfaces of revolution
 - 58-07 **Cylindrical and Spherical Coordinates**
 - 58-99 Associated problems in Chapter 58

- 59 Vector-Valued Functions**
 - 59-01 **Vector-Valued Functions** [] space curves, limits and continuity
 - 59-02 **Differentiation and Integration of Vector-Valued Functions**
 - 59-03 **Velocity and Acceleration** [] projectile motion
 - 59-04 **Tangent Vectors and Normal Vectors** [] tangential, normal components of acceleration
 - 59-05 **Arc Length and Curvature** [] arc length parameter, applications
 - 59-06 **Motion in Space: Velocity and Acceleration**
 - 59-99 Associated problems in Chapter 59

- 60 Functions of Several Variables**
 - 60-01 **Introduction to Functions of Several Variables** [] graphs, level curves and surfaces
 - 60-02 **Limits and Continuity** [] neighborhoods in the plane, limit and continuity of a function of two variables, continuity of a function of three variables
 - 60-03 **Partial Derivatives** [] two or more variables, higher order partial derivatives
 - 60-04 **Differentials** [] increments, differentiability, approximation
 - 60-05 **Chain Rules for Functions of Several Variables** [] implicit partial derivatives
 - 60-06 **Directional Derivatives and Gradients** [] applications, functions of three variables
 - 60-07 **Tangent Planes and Normal Lines** [] angle of inclination, comparison of gradients $\nabla f(x, y)$ and $\nabla F(x, y, z)$
 - 60-08 **Extrema of Functions of Two Variables** [] absolute & relative extrema, second partials test
 - 60-09 **Applications of Extrema of Functions of Two Variables** [] applied optimization problems, method of least squares
 - 60-10 **Lagrange Multipliers** [] constrained optimization problems, Lagrange multipliers with two constraints
 - 60-99 Associated problems in Chapter 60

- 61 Multiple Integration**

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- 61-01 **Iterated Integrals and Area in the Plane**
- 61-02 **Double Integrals and Volume** □ properties, evaluation
- 61-03 **Change of Variables: Polar Coordinates** change of variables to polar form
- 61-04 **Center of Mass and Moments of Inertia**
- 61-05 **Surface Area**
- 61-06 **Triple Integrals and Applications** □ CM and moments of inertia
- 61-07 **Triple Integrals in Cylindrical and Spherical Coordinates**
- 61-08 **Change of Variables: Jacobians**
- 61-99 Associated problems in Chapter 61

62 Vector Analysis

- 62-01 **Vector Fields** □ conservative vector fields; curl, divergence of vector fields
- 62-02 **Line Integrals** □ piecewise smooth curves, vector fields, differential form
- 62-03 **Conservative Vector Fields and Independence of Path** □ fundamental theorem of line integrals, energy conservation
- 62-04 **Green's Theorem** □ alternative forms
- 62-05 **Parametric Surfaces** □ find equations, normal vectors, tangent planes, area of surface
- 62-06 **Surface Integrals** □ parametric surfaces, orientation of surface, flux integrals
- 62-07 **Divergence Theorem** □ flux
- 62-08 **Stokes's Theorem** □ physical interpretation of curl
- 62-99 Associated problems in Chapter 62

63 Higher Order Differential Equations

- 63-01 **Second-Order Homogeneous Linear DEs** □ $P(x) \frac{d^2 y}{dx^2} + Q(x) \frac{dy}{dx} + R(x) y = 0$
- 63-02 **Second-Order Nonhomogeneous Linear DEs** □ undetermined coefficients, variation of parameters
- 63-03 **Series Solutions** □ power series solution, approximation by Taylor series
- 63-04 **Systems of DEs**
- 63-05 **Applications of Second-Order DEs** □ vibrating springs, electric circuits
- 63-99 Associated problems in Chapter 63

63 Continuous Probability

- 63-01 **Probability**
- 63-99 Associated problems in Chapter 63