ALGORITHMS FOR MIXED INTEGER PROGRAMMING
(ORI 391Q.6)

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PREREQUISITES: Graduate courses in linear programming and integer programming; a working knowledge of at least one computer language.

TEXTS Required: G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley & Sons, New York, 1988. (Out of print. Try Internet.)

Recommended: D. L. Applegate, R. E. Bixby, V. Chvátal and W. J. Cook *The Traveling Salesman Problem: A Computational Study*, Princeton University Press, Princeton, NJ, 2006.

E. L. Lenstra, A. H. G. Rinnooy Kan, and D. B. Shmoys, *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*, John Wiley & Sons, New York, 1984.

OBJECTIVE: Over the last few years great strides have been made in providing optimal or near-optimal solutions to large-scale mixed integer programming (MIP) problems. The aim of this course is to investigate many of the techniques that have been developed for this purpose. These include the use of intelligent heuristics, decomposition and column generation techniques, polyhedral theory, as well as ad hoc procedures. In fact, it is rare that any one technique can be applied successfully to solve MIPs that arise in practice. What is needed is a strategy that combines insights about a particular problem with lower bounding procedures, limited enumeration, and simple methods for quickly finding good feasible solutions. Examples taken from industry will serve as a backdrop to the class discussion. Emphasis will be placed on the development of computational software.

GRADING:

Homework	-	10%
Programs	-	10%
Project	-	25%
Midterm Exam	-	25%
Final Exam	-	30%

NOTE: Homework that is one class late will be penalized 10%; it will not be accepted after that date. All students must take the exams when scheduled. There will be **NO** make-up exams and there will be **NO** incompletes in the course; every student will get a grade.

If you would like to review your final exam, please do so no later than one week after the next semester begins (fall, spring, or summer). After that time, all exams and other retrieved class material will be discarded.

EXTRA CREDIT: There will be **NO** extra assignments for those wishing to try to improve their overall grade.

DISHONESTY: University policies for academic dishonesty will be strictly followed. Students found cheating on any exam will receive a grade of "F" in the course. Homework and other assignments turned in that do not represent the student's original work will receive a grade of zero.

DISABILITIES: The University of Texas provides upon request academic adjustments for students with documented disabilities. For more information, contact the Division of Diversity and Community Engagement, Services for Students with Disabilities, 512-471-6259, http://diversity.utexas.edu/disability/.

Course Schedule

Chapter	*Pages*	*Topic*	*Assignments*
I.1	3-20	Introduction Modeling Preprocessing [4] Computational complexity Commercial codes	HW #1 (Railcar pumps) Due: September 10
I.2 II.3 II.5	41-49 323-337 409-412	Lagrangian Relaxation Power plant scheduling [7] Tool switching on an FMS [6] Bundle method [16], [19] Generating multipliers [8], [11] Facility location [13]	HW #2 from N&W page 346, Section II.4 (see below): Exercises 13 (use Eq. (6.1) to find optimal multiplier values), 14, 16 (first objective function term should be preceded by a "–" sign; see note in problem statement below) Due: September 24
II.3 II.5	337-341 412-417	Benders Decomposition Equipment selection [47]	HW #3 (handout- Benders) Due: October 15
†		Column Generation (D-W Decomposition) Vehicle routing [33], [36] Machine scheduling [31]	HW #4 (handout – ODIMCF) Due: October 29 HW #5 (handout – DW Decomp) Due: November 3
I.4 II.1 II.2	83-109 205-217 259-290	Polyhedral Theory - Valid Inequalities General integer programs [46] TSP [44], [48] Crew Scheduling [49]	HW #6 from N&W page 109, Section I.4 (see below): Exercises 1, 2. Page 291, Section II.2: Exercise 3 [part i) should be = $mn + n - m$] Due: November 24
II.5	407-409	Heuristics (GRASP, smoothing) Vehicle routing [25] American Airlines [26] Smoothing [56] Tabu search vs. GRASP [28]	
		General Decomposition Personnel scheduling [55]	

†Chapter 11, L.A. Wolsey, Integer Programming, Wiley & Sons, 1998.

References

Introduction

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Bundle Method

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GRASP - Greedy Randomized Adaptive Search Procedure

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Column Generation Techniques -- Branch and Price

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Computations

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