SIGNIFICANT FIGURES

What are significant figures?

All measurements have some degree of uncertainty; how great the uncertainty is depends on both the accuracy of the measuring device and the skill of its operator. On a triple-beam platform balance, the mass of a sample substance can be measured to the nearest 0.1 g; mass differences less than this cannot be detected on this balance. We might therefore indicate the mass of a dime measured on this balance as 2.2 $\pm$ 0.1 g. The $\pm$0.1 (read plus or minus 0.1) is a measure of the accuracy of the measurement. It is important to have some indication of how accurately any measurement is made; the $\pm$ notation is one way to accomplish this. It is common to drop the $\pm$ notation with the understanding that *there is uncertainty of at least one unit in the last digit of the measured quantity*. That is, measured quantities are reported in such a way that only the last digit is uncertain. All of the digits, including the uncertain one, are called significant digits or, more commonly, significant figures. The number 2.2 has two significant figures, while the number 2.2405 has five significant figures.

How can we determine how many significant numbers a measurement has?

The following rules apply when determining the number of significant figures in a measured quantity:

1. All nonzero digits are significant: 457 cm (three significant figures); 0.25 g (two significant figures).
2. Zeros between nonzero digits are significant: 1005 kg (four significant figures); 1.03 cm (three significant figures).
3. Zeros to the left of the first nonzero digits in a number are not significant, they merely indicate the position of the decimal point: 0.02 g (one significant figure); 0.0026 cm (two significant figures).
4. When a number ends in zeros that are to the right of the decimal point, they are significant: 0.0200 g (three significant figures); 3.0 cm (two significant figures).
5. When a number ends in zeros that are not to the right of a decimal point, the zeros are not necessarily significant: 130 cm (two or three significant figures); 10,300 g (three, four, or five significant figures). The way to remove this ambiguity is described below.

Use of standard exponential notation avoids the potential ambiguity of whether the zeros at the end of a number are significant (rule 5). For example, a mass of 10,300 g can be written in exponential notation showing three, four, or five significant figures:

1.03×10^4 g (three significant figures)
1.030×10^4 g (four significant figures)
1.0300×10^4 g (five significant figures)

In these numbers, all the zeros to the right of the decimal point are significant (rules 2 and 4).

How do we work with significant figures? In carrying measured quantities through calculations, the rule used is that the accuracy of the result is limited by the least accurate measurement.

Addition and Subtraction

When adding or subtracting, the number of digits to the right of the decimal point in the answer is determined by the measurement which has the least number of digits to the right of the decimal point.

e.g., adding:

$$\begin{array}{rl} 26.46 & \leftarrow \text{ this has the least digits to the right of the decimal point (2)} \\ \underline{+\ 4.123} & \\ 30.583 & \text{- rounds off to } 30.58 \leftarrow \text{ 2 digits to the right of the decimal point} \end{array}$$

(When rounding off, do the following: When the number to be dropped is less than 5, it is just dropped [e.g., 6.34 rounds off to 6.3]. When it is more than 5, the preceding number is increased by 1 [e.g., 5.27 rounds off to 5.3]. When the number to be dropped is 5, the preceding number is not changed when it [the preceding number] is even [i.e., 4.45 rounds off to 4.4]. When the preceding number is odd, it is increased by 1 [i.e., 4.35 rounds off to 4.4]. In fairness, we must note that the even/odd rules for rounding terminal 5's are sometimes ignored; instead, the preceding number is increased by 1 when a 5 is dropped.)

e.g., subtracting:

$$\begin{array}{rl} 26.46 & \\ \underline{-\ 4.123} & \\ 22.337 & \text{- rounds off to } 22.34 \end{array}$$

The above rule is based on the fact that the last digit retained in the sum or difference is determined by the first doubtful figure (which is underlined in the following example).

$$\begin{array}{rl} 37.2\underline{4} & \\ \underline{+10.3\ \ } & \\ 47.\underline{54} & \text{- rounds off to } 47.\underline{5} \end{array}$$

We may report the first doubtful figure, but no more.

Multiplication and Division

In multiplying or dividing, the number of significant figures in the answer--regardless of the position of the decimal point--equals that of the quantity which has the smaller number of significant figures.

e.g., multiplying:

$$\begin{array}{rl} 2.61 & \\ \underline{\times\ 1.2\ \ } & \text{- this has the smaller number of significant figures (2)} \\ 3.132 & \text{- rounds off to 3.1; has 2 significant figures} \end{array}$$

e.g., dividing: $2.61 \div 1.2 = 2.175$ - rounds off to 2.2

Again, this rule has been based on the fact that we may report only one doubtful figure. For example, if we underline each uncertain figure as well as each figure obtained from an uncertain figure, the step-by-step multiplication gives:

$$\begin{array}{rl} 12.34 & \\ \underline{\times\ 1.23} & \leftarrow \text{ this has the least number of significant figures (3)} \\ \underline{3702} & \\ 246\underline{8}\ & \\ \underline{1234\ \ \ \ } & \\ 15.\underline{1782} & = 15.\underline{2} \text{ - the answer must have 3 significant figures.} \end{array}$$

This handout has used material from the following sources: Chemistry the Central Science, Brown and LeMay, Prentice Hall, 1977; Determining Significant Figures, Series 800. Basic Math for Science Students and Technicians, Prentice Hall Media, 1972.

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