The University of Texas at Austin
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Differentiation Formulas, Rules, and Examples

Rules

Here, k is a constant, and f, g, and h are differentiable functions.

1. Derivative of a constant:
$$\frac{\mathrm{d}}{\mathrm{d}x}[k] = 0$$
2. Constant-multiple Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}[kf(x)] = kf'(x)$$
3. Sum-difference Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$
4. Power Rule
$$\frac{\mathrm{d}}{\mathrm{d}x}[x^k] = kx^{k-1}$$
5. Product Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$
6. Product Rule (three functions):
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)h(x)] = fgh' + fg'h + f'gh$$
7. Quotient Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$$
8. Reciprocal Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{(g(x))^2}$$
9. Chain Rule:
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Derivatives of Common Functions

Here, a is a constant.

1. $\frac{\mathrm{d}}{\mathrm{d}x}[e^x] = e^x$
2. $\frac{\mathrm{d}}{\mathrm{d}x}[\ln x] = \frac{1}{x}, \quad x > 0$
3. $\frac{\mathrm{d}}{\mathrm{d}x}[a^x] = (\ln a) \cdot a^x$
4. $\frac{\mathrm{d}}{\mathrm{d}x}[\log_a x] = \frac{1}{x \ln a}$
5. $\frac{\mathrm{d}}{\mathrm{d}x}[\sin x] = \cos x$
6. $\frac{\mathrm{d}}{\mathrm{d}x}[\cos x] = -\sin x$
7. $\frac{\mathrm{d}}{\mathrm{d}x}[\tan x] = \sec^2 x$
8. $\frac{\mathrm{d}}{\mathrm{d}x}[\sec x] = \sec x \tan x$
9. $\frac{\mathrm{d}}{\mathrm{d}x}[\csc x] = -\csc x \cot x$
10. $\frac{\mathrm{d}}{\mathrm{d}x}[\cot x] = -\csc^2 x$
11. $\frac{\mathrm{d}}{\mathrm{d}x}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}, |x| < 1$
12. $\frac{\mathrm{d}}{\mathrm{d}x}[\tan^{-1} x] = \frac{1}{1+x^2}$
13. $\frac{\mathrm{d}}{\mathrm{d}x}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$
14. $\frac{\mathrm{d}}{\mathrm{d}x}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}, |x| < 1$
15. $\frac{\mathrm{d}}{\mathrm{d}x}[\cot^{-1} x] = -\frac{1}{1+x^2}$
16. $\frac{\mathrm{d}}{\mathrm{d}x}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}, |x| > 1$

The conditions on $|x|$ in (11), (13), (14), and (16) are so that the values under the square roots are positive.

Example 1. Using the Product Rule, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}[e^x \cos x] = e^x \frac{\mathrm{d}}{\mathrm{d}x}[\cos x] + \frac{\mathrm{d}}{\mathrm{d}x}[e^x] \cos x = -e^x \sin x + e^x \cos x \,.$$

Example 2. Using the Chain Rule, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}\left(\frac{x}{2}\right)\right] = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x}{2}\right] = \frac{\frac{1}{2}}{1+\frac{x^2}{4}} = \frac{2}{4+x^2} \,.$$

To obtain the last equality, multiply the numerator and denominator by 4.

Example 3. Using the Product Rule for three functions, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}[xe^x \cos x] = xe^x(-\sin x) + x(e^x)\cos x + (1)e^x \cos x = -xe^x \sin x + xe^x \cos x + e^x \cos x \,.$$

Example 4. We can use the Quotient Rule to get the derivative of $\tan x$ as follows:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}[\tan x] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\sin x}{\cos x}\right] &= \frac{(\cos x)\cos x - (-\sin x)\sin x}{(\cos x)^2} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \,.\end{aligned}$$

Example 5. Using the Chain Rule, we compute

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{x^3}\right] = \left(e^{x^3}\right)\left(\frac{\mathrm{d}}{\mathrm{d}x}\left[x^3\right]\right) = 3x^2 e^{x^3} \,.$$

Note that the derivative of e^x is just e^x, so that's why e^{x^3} doesn't appear to change here.

Example 6. Again, using the Chain Rule gives us

$$\frac{\mathrm{d}}{\mathrm{d}x}[\cos(e^x)] = (-\sin(e^x))(e^x) = -e^x \sin(e^x) \,.$$