Integration Formulas and Examples

$f(x)$ and $g(x)$ are integrable functions, and a is a constant.

1. $\displaystyle\int x^a \,\mathrm{d}x = \frac{x^{a+1}}{a+1} + C$
2. $\displaystyle\int af(x) \,\mathrm{d}x = a\int f(x) \,\mathrm{d}x$
3. $\displaystyle\int f \pm g \,\mathrm{d}x = \int f \,\mathrm{d}x \pm \int g \,\mathrm{d}x$
4. $\displaystyle\int e^x \,\mathrm{d}x = e^x + C$
5. $\displaystyle\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$
6. $\displaystyle\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + C$
7. $\displaystyle\int \cos x \,\mathrm{d}x = \sin x + C$
8. $\displaystyle\int \sin x \,\mathrm{d}x = -\cos x + C$
9. $\displaystyle\int \sec^2 x \,\mathrm{d}x = \tan x + C$
10. $\displaystyle\int \csc^2 x \,\mathrm{d}x = -\cot x + C$
11. $\displaystyle\int \sec x \tan x \,\mathrm{d}x = \sec x + C$
12. $\displaystyle\int \csc x \cot x \,\mathrm{d}x = -\csc x + C$
13. $\displaystyle\int \tan x \,\mathrm{d}x = \ln|\sec x| + C$
14. $\displaystyle\int \cot x \,\mathrm{d}x = \ln|\sin x| + C$
15. $\displaystyle\int \sec x \,\mathrm{d}x = \ln|\sec x + \tan x| + C$
16. $\displaystyle\int \csc x \,\mathrm{d}x = \ln|\csc x - \cot x| + C$
17. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
18. $\displaystyle\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
19. $\displaystyle\int \frac{\mathrm{d}x}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
20. $\displaystyle\int \ln x \,\mathrm{d}x = x\ln x - x + C$
21. $\displaystyle\int \sin^2 x \,\mathrm{d}x = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
22. $\displaystyle\int \cos^2 x \,\mathrm{d}x = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$
23. $\displaystyle\int \tan^2 x \,\mathrm{d}x = \tan x - x + C$
24. $\displaystyle\int \cot^2 x \,\mathrm{d}x = -\cot x - x + C$

Example 1. We can split up $\int 5x^3 - 2x^2 \, \mathrm{d}x$ to get

$$\int 5x^3 - 2x^2 \, \mathrm{d}x = \int 5x^3 \, \mathrm{d}x - \int 2x^2 \, \mathrm{d}x \, ,$$

and then we pull out the constants to get

$$\int 5x^3 \, \mathrm{d}x - \int 2x^2 \, \mathrm{d}x = 5 \int x^3 \, \mathrm{d}x - 2 \int x^2 \, \mathrm{d}x \, .$$

Finally, integrating gives

$$5 \int x^3 \, \mathrm{d}x - 2 \int x^2 \, \mathrm{d}x = 5 \left(\frac{x^4}{4} \right) - 2 \left(\frac{x^3}{3} \right) + C = \frac{5x^4}{4} - \frac{2x^3}{3} + C \, .$$

Example 2. Split up $\int e^x - \cos x + \frac{1}{x} \, \mathrm{d}x$ to get

$$\int e^x - \cos x + \frac{1}{x} \, \mathrm{d}x = \int e^x \, \mathrm{d}x - \int \cos x \, \mathrm{d}x + \int \frac{1}{x} \, \mathrm{d}x \, ,$$

which gives us

$$\int e^x \, \mathrm{d}x - \int \cos x \, \mathrm{d}x + \int \frac{1}{x} \, \mathrm{d}x = e^x - \sin x + \ln |x| + C \, .$$

Example 3. Pull out the constant from $\int \frac{3 \, \mathrm{d}x}{9 + x^2}$ to get

$$\int \frac{3 \, \mathrm{d}x}{9 + x^2} = 3 \int \frac{\mathrm{d}x}{3^2 + x^2} \, .$$

Integrating this gives

$$3 \int \frac{\mathrm{d}x}{3^2 + x^2} = 3 \left(\frac{1}{3} \tan^{-1} \left(\frac{x}{3} \right) \right) + C = \tan^{-1} \left(\frac{x}{3} \right) + C \, .$$