Trigonometric Identities and the Unit Circle

1. **Negative Angle Identities**

$$\sin(-\theta) = -\sin\theta \quad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \quad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \quad \cot(-\theta) = -\cot\theta$$

2. **Addition and Subtraction Identities**

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

3. **Double Angle Identities**

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

4. **Product Identities**

$$\sin A \cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$
$$\cos A \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$$
$$\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$$
$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

5. **Cofunction Identities**

$$\sin A = \cos\left(\frac{\pi}{2} - A\right)$$
$$\sec A = \csc\left(\frac{\pi}{2} - A\right)$$
$$\tan A = \cot\left(\frac{\pi}{2} - A\right)$$

6. **Quotient Identities**

$$\csc\theta = \frac{1}{\sin\theta} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

7. **Pythagorean Identities**

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

8. **Half-Angle Identities**

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$
$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$
$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

9. **Sum Identities**
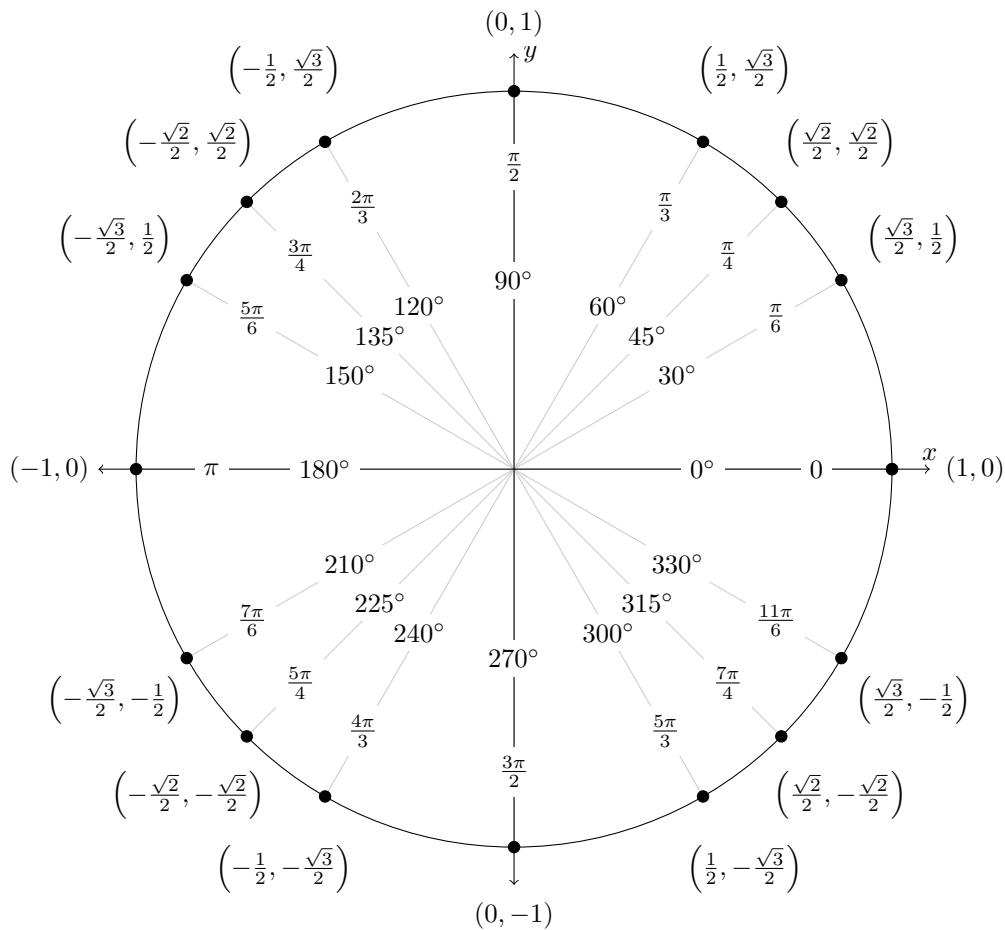
$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
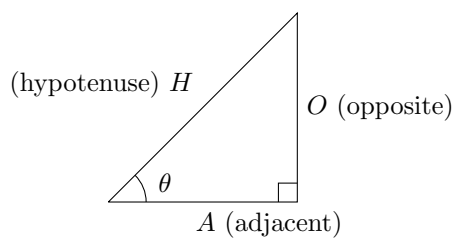$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$(0, 1)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{\pi}{2}$
$\frac{2\pi}{3}$ $\frac{\pi}{3}$
$\frac{3\pi}{4}$ $\frac{\pi}{4}$
$90°$ $\frac{5\pi}{6}$ $\frac{\pi}{6}$
$120°$ $60°$
$135°$ $45°$
$150°$ $30°$

$(-1, 0)$ ← π — $180°$ —— $0°$ —— 0 → x $(1, 0)$

$210°$ $330°$
$225°$ $315°$
$\frac{7\pi}{6}$ $240°$ $300°$ $\frac{11\pi}{6}$
$\frac{5\pi}{4}$ $270°$ $\frac{7\pi}{4}$
$\frac{4\pi}{3}$ $\frac{3\pi}{2}$ $\frac{5\pi}{3}$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$(0, -1)$

Trigonometric Definitions

(hypotenuse) H

O (opposite)

θ

A (adjacent)

SOH	$\sin\theta = O/H$
CAH	$\cos\theta = A/H$
TOA	$\tan\theta = O/A.$

Special Angle Triangles

$2x$

x

$30°$

$x\sqrt{3}$

$x\sqrt{2}$

x

$45°$

x

2