Vector loop:

$$\bar{R}_2 + \bar{R}_3 - \bar{R}_4 - \bar{R}_5 - \bar{R}_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_5} - fe^{j\cancelto{0}{\theta_1}} = 0$$

$$= a(\cos\theta_2 + j\sin\theta_2) + b(\cos\overset{?}{\theta_3} + j\sin\overset{?}{\theta_3}) - c(\cos\overset{?}{\theta_4} + j\sin\overset{?}{\theta_4}) - d(\cos\theta_5 + j\sin\theta_5) - f = 0$$

Solve for θ_4:

Real: $a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_5 - f = 0$

$$b\cos\theta_3 = -a\cos\theta_2 + c\cos\theta_4 + d\cos\theta_5 + f$$

$$b^2\cos^2\theta_3 = (-a\cos\theta_2 + c\cos\theta_4 + d\cos\theta_5 + f)^2 \quad ①$$

Img: $a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 - d\sin\theta_5 = 0$

$$b\sin\theta_3 = -a\sin\theta_2 + c\sin\theta_4 + d\sin\theta_5$$

$$b^2\sin^2\theta_3 = (-a\sin\theta_2 + c\sin\theta_4 + d\sin\theta_5)^2 \quad ②$$

① + ② : Eliminate θ_3

$$b^2\cancelto{1}{(\sin^2\theta_3 + \cos^2\theta_3)} = (-a\cos\theta_2 + c\cos\theta_4 + d\cos\theta_5 + f)^2 + (-a\sin\theta_2 + c\sin\theta_4 + d\sin\theta_5)^2$$

$$b^2 = a^2\cancelto{1}{(\sin^2\theta_2 + \cos^2\theta_2)} - 2ac(\sin\theta_2\sin\theta_4 + \cos\theta_2\cos\theta_4) - 2ad(\sin\theta_2\sin\theta_5 + \cos\theta_2\cos\theta_5)$$
$$- 2af\cos\theta_2 + c^2\cancelto{1}{(\sin^2\theta_4 + \cos^2\theta_4)} + 2cd(\sin\theta_4\sin\theta_5 + \cos\theta_4\cos\theta_5) + 2cf\cos\theta_4$$
$$+ d^2\cancelto{1}{(\sin^2\theta_5 + \cos^2\theta_5)} + 2df\cos\theta_5 + f^2$$

Half-Angle Identity: $\sin\theta_4 = \dfrac{2\tan\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)} \qquad \cos\theta_4 = \dfrac{1-\tan^2\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)}$

$$\theta_5 = \lambda\theta_2 + \phi$$

$$A = 2c(d\cos\theta_5 - a\cos\theta_2 + f) = 2c(d\cos(\lambda\theta_2 + \phi) - a\cos\theta_2 + f)$$

$$B = 2c(d\sin\theta_5 - a\sin\theta_2) = 2c(d\sin(\lambda\theta_2 + \phi) - a\sin\theta_2)$$

$$C = a^2 - b^2 + c^2 + d^2 + f^2 - 2af\cos\theta_2 - 2d(a\cos\theta_2 - f)\cos(\lambda\theta_2 + \phi)$$
$$- 2ad\sin\theta_2\sin(\lambda\theta_2 + \phi)$$

$D = C - A$

$E = 2B$ $\qquad \Rightarrow \quad \theta_{4_{1,2}} = 2\arctan\left(\dfrac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)$

$F = A + C$

System always operates as if θ_4 is <u>crossed</u>

$$\Rightarrow \boxed{\theta_4 = \theta_{4_2} = 2\arctan\left(\frac{-E - \sqrt{E^2 - 4DF}}{2D}\right)}$$

Solve for θ_3:

Real: $a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_5 - f = 0$

$$c\cos\theta_4 = a\cos\theta_2 + b\cos\theta_3 - d\cos\theta_5 - f$$

$$c^2\cos^2\theta_4 = (a\cos\theta_2 + b\cos\theta_3 - d\cos\theta_5 - f)^2 \quad (3)$$

Img: $a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 - d\sin\theta_5 = 0$

$$c\sin\theta_4 = a\sin\theta_2 + b\sin\theta_3 - d\sin\theta_5$$

$$c^2\sin^2\theta_4 = (a\sin\theta_2 + b\sin\theta_3 - d\sin\theta_5)^2 \quad (4)$$

(3) + (4) : Eliminate θ_4

$$c^2(\underbrace{\cos^2\theta_4 + \sin^2\theta_4}_{1}) = (a\cos\theta_2 + b\cos\theta_3 - d\cos\theta_5 - f)^2 + (a\sin\theta_2 + b\sin\theta_3 - d\sin\theta_5)^2$$

$$c^2 = a^2 + 2ab(\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3) - 2ad(\sin\theta_2\sin\theta_5 + \cos\theta_2\cos\theta_5)$$
$$+ 2af\cos\theta_2 + b^2 - 2bd(\sin\theta_3\sin\theta_5 + \cos\theta_3\cos\theta_5) + 2bf\cos\theta_3$$
$$+ d^2 - 2df\cos\theta_5 + f^2$$

Half Angle Identity: $\sin\theta_4 = \dfrac{2\tan\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)}$ $\quad \cos\theta_4 = \dfrac{1-\tan^2\left(\frac{\theta_4}{2}\right)}{1+\tan^2\left(\frac{\theta_4}{2}\right)}$

$$G = 2b(a\cos\theta_2 - d\cos\theta_5 - f) = 2b(a\cos\theta_2 - d\cos(\lambda\theta_2 + \phi) - f)$$
$$H = 2b(a\sin\theta_2 - d\sin(\lambda\theta_2 + \phi))$$

$$K = a^2 + b^2 - c^2 + d^2 + f^2 - 2af\cos\theta_2 - 2d(a\cos\theta_2 - f)\cos(\lambda\theta_2 + \phi)$$
$$- 2ad\sin\theta_2\sin(\lambda\theta_2 + \phi)$$

$L = K - G$

$M = 2H$

$N = G + K$

$$\theta_{3_{1,2}} = 2\arctan\left(\frac{-M \pm \sqrt{M^2 - 4LN}}{2L}\right)$$

System always operates as if θ_3 is open

$$\Rightarrow \boxed{\theta_3 = \theta_{3_1} = 2\arctan\left(\frac{-M + \sqrt{M^2 - 4LN}}{2L}\right)}$$

Location of point P:

$$P = \bar{R}_1 + \bar{R}_{PA} = a(\cos\theta_2 + j\sin\theta_2) + p(\cos(\theta_3 + \delta) + j\sin(\theta_3 + \delta))$$

$$\boxed{\begin{aligned} P_x &= a\cos\theta_2 + p\cos(\theta_3 + \delta) \\ P_y &= a\sin\theta_2 + p\sin(\theta_3 + \delta) \end{aligned}}$$

Velocity Analysis

Same vector loop:

$$\bar{R}_2 + \bar{R}_3 - \bar{R}_4 - \bar{R}_5 - \bar{R}_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_5} - fe^{\cancelto{0}{j\theta_1}} = 0$$

$$\frac{d}{dt} = a\omega_2 je^{j\theta_2} + b\omega_3 je^{j\theta_3} - c\omega_4 je^{j\theta_4} - d\omega_5 je^{j\theta_5} = 0$$

$$= a\omega_2 j(\cos\theta_2 + j\sin\theta_2) + b\omega_3 j(\cos\overset{?}{\theta_3} + j\sin\theta_3) - c\omega_4 j(\cos\overset{?}{\theta_4} + j\sin\theta_4) - d\omega_5 j(\cos\theta_5 + j\sin\theta_5) = 0$$

$$= a\omega_2(-\sin\theta_2 + j\cos\theta_2) + b\omega_3(-\sin\theta_3 + j\cos\theta_3) - c\omega_4(-\sin\theta_4 + j\cos\theta_4) - d\omega_5(-\sin\theta_5 + j\cos\theta_5) = 0$$

From before: $\theta_5 = \lambda\theta_2 + \phi$

$$\frac{d}{dt} = \omega_5 = \lambda\omega_2$$

Real: $-a\omega_2\sin\theta_2 - b\omega_3\sin\theta_3 + c\omega_4\sin\theta_4 + d\omega_5\sin\theta_5 = 0$

$$\omega_3 = \frac{a\omega_2\sin\theta_2 - c\omega_4\sin\theta_4 - d\omega_5\sin\theta_5}{b\sin\theta_3} \quad (1)$$

Img: $a\omega_2\cos\theta_2 + b\omega_3\cos\theta_3 - c\omega_4\sin\theta_4 - d\omega_5\sin\theta_5 = 0 \quad (2)$

(1) sub into (2)

$$\Rightarrow \boxed{\omega_3 = \frac{-2\sin\theta_4\left(a\omega_2\sin(\theta_2-\theta_4) + d\omega_5\sin(\theta_4-\theta_5)\right)}{b\left(\cos(\theta_3-2\theta_4) - \cos\theta_3\right)}}$$

$$\boxed{\omega_4 = \frac{a\omega_2\sin\theta_2 + b\omega_3\sin\theta_3 - d\omega_5\sin\theta_5}{c\sin\theta_4}}$$

Linear velocity of point P:

$$V_P = \bar{V}_1 + \bar{V}_{PA} = a\omega_2(-\sin\theta_2 + j\cos\theta_2) + p\omega_3(-\sin(\theta_3+\delta) + j\cos(\theta_3+\delta))$$

$$\boxed{V_x = -a\omega_2\sin\theta_2 - p\omega_3\sin(\theta_3+\delta)}$$

$$\boxed{V_y = a\omega_2\cos\theta_2 + p\omega_3\cos(\theta_3+\delta)}$$

Acceleration Analysis

Same vector loop:

$$\bar{R}_2 + \bar{R}_3 - \bar{R}_4 - \bar{R}_5 - \bar{R}_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_5} - fe^{j\cancelto{0}{\theta_1}} = 0$$

$$\frac{d}{dt} = a\omega_2 je^{j\theta_2} + b\omega_3 je^{j\theta_3} - c\omega_4 je^{j\theta_4} - d\omega_5 je^{j\theta_5} = 0$$ time-dependent

$$\frac{d^2}{dt^2} = a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2} + b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}$$
$$- c\alpha_4 je^{j\theta_4} + c\omega_4^2 e^{j\theta_4} - d\alpha_5 je^{j\theta_5} + d\omega_5^2 e^{j\theta_5} = 0$$

$$= a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2) + b\alpha_3(-\sin\theta_3 + j\cos\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3)$$
$$- c\alpha_4(-\sin\theta_4 + j\cos\theta_4) + c\omega_4^2(\cos\theta_4 + j\sin\theta_4) - d\alpha_5(-\sin\theta_5 + j\cos\theta_5) + d\omega_5^2(\cos\theta_5 + j\sin\theta_5) = 0$$

$$\theta_5 = \lambda\theta_2 + \phi$$
$$\frac{d}{dt} = \omega_5 = \lambda\omega_2$$
$$\frac{d^2}{dt^2} = \alpha_5 = \lambda\alpha_2$$

Real: $-a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 - b\alpha_3\sin\theta_3 - b\omega_3^2\cos\theta_3$
$+ c\alpha_4\sin\theta_4 + c\omega_4^2\cos\theta_4 + d\alpha_5\sin\theta_5 + d\omega_5^2\cos\theta_5 = 0$

Img: $a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 + b\alpha_3\cos\theta_3 - b\omega_3^2\sin\theta_3$
$- c\alpha_4\cos\theta_4 + c\omega_4^2\sin\theta_4 - d\alpha_5\cos\theta_5 + d\omega_5^2\sin\theta_5 = 0$

$$\Rightarrow \alpha_3 = \frac{-a\alpha_2\sin(\theta_2-\theta_4) - a\omega_2^2\cos(\theta_2-\theta_4) - b\omega_3^2\cos(\theta_3-\theta_4) + d\omega_5^2\cos(\theta_5-\theta_4) + d\alpha_5\sin(\theta_5-\theta_4) + c\omega_4^2}{b\sin(\theta_3-\theta_4)}$$

$$\alpha_4 = \frac{a\alpha_2\sin(\theta_2-\theta_3) + a\omega_2^2\cos(\theta_2-\theta_3) - c\omega_4^2\cos(\theta_3-\theta_4) - d\omega_5^2\cos(\theta_3-\theta_5) + d\alpha_5\sin(\theta_3-\theta_5) + b\omega_3^2}{c\sin(\theta_4-\theta_3)}$$

Acceleration of point P:

$$A_P = \bar{A}_1 + \bar{A}_{PA} = a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)$$
$$+ p\alpha_3(-\sin(\theta_3+\delta) + j\cos(\theta_3+\delta)) - p\omega_3^2(\cos(\theta_3+\delta) + j\sin(\theta_3+\delta))$$

$$A_x = -a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 - p\alpha_3\sin(\theta_3+\delta) - p\omega_3^2\cos(\theta_3+\delta)$$
$$A_y = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 + p\alpha_3\cos(\theta_3+\delta) - p\omega_3^2\sin(\theta_3+\delta)$$