The Fern Algorithm for Real-Number Coding

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The Problem

How do I code real numbers ...

- compactly?
- conveniently?
- in a form conducive to decision-making?

Known Coding Schemes

- Numerals

$$\pi = 3(10^0) + 1(10^{-1}) + 4(10^{-2}) + 1(10^{-3}) + 5(10^{-4}) + 9(10^{-5})$$
$$= 3.14159$$

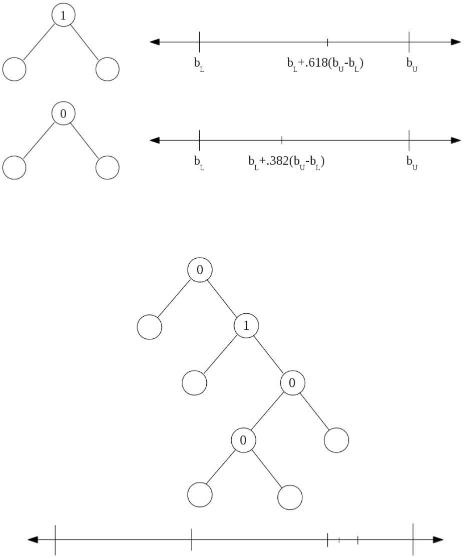
- Counting

$$\pi = 1(1) + 1(1) + 1(1) = |||$$

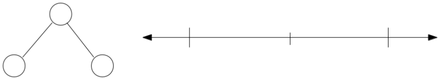
- Ratios

$$\pi = \frac{\text{卌 卌 卌 卌 卌 卌 |}}{\text{卌 卌}}$$

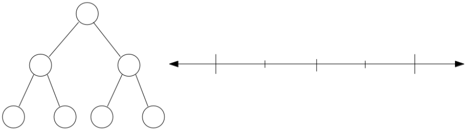
Fern Coding...



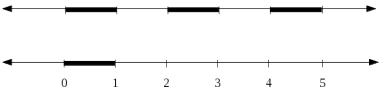
...Compared to Binary Numerals



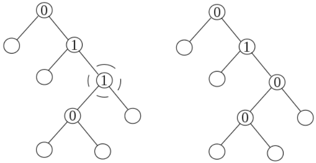
(a) One-digit binary number



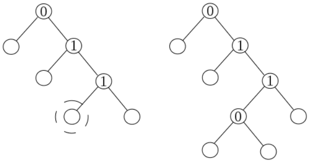
(b) Two-digit binary number



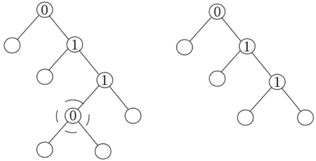
Genetic Mutation



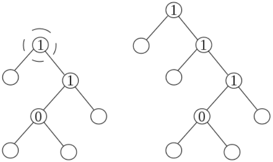
(c) Flipping a Fork Bit



(d) Splitting a Leaf

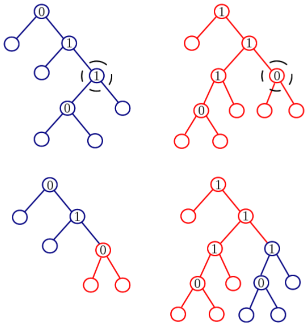


(e) Merging a Subtree into a Leaf

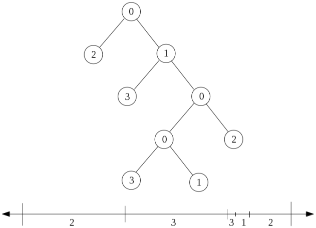


(f) Adding a New Root

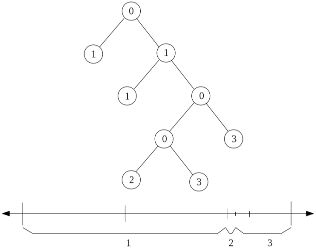
Genetic Crossover



Fern as a Classifier

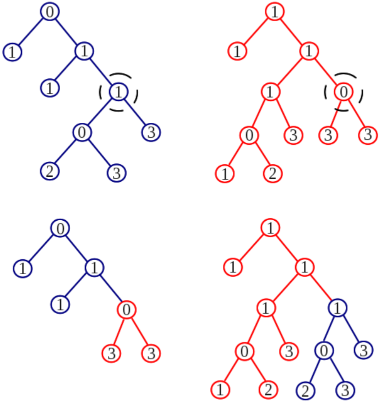


(g) Unconstrained Binning

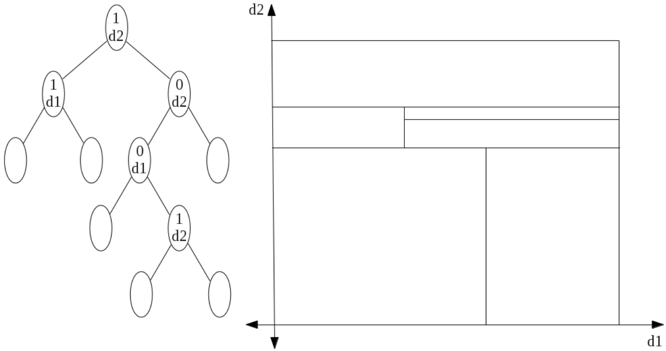


(h) Consecutive Binning

Semantic Conflicts with Genetic Crossover



n-Dimensional Quantities



Test Case: Attitude Control of a Satellite

State Equations:

$$\dot{\theta} = \frac{h}{J}$$

$$\dot{h} = U$$

Optimal Solution as a function of time:

$$U(t) = \begin{cases} (-u_0)sign(\theta_0) & t \leq t_s \\ (u_0)sign(\theta_0) & t_s < t < t_f \\ 0 & t \geq t_f \end{cases}$$

$$t_s = \frac{h_0}{u_0} + \frac{J}{2u_0}\sqrt{2\frac{h_0^2}{J^2} + 4\frac{u_0}{J}\theta_0}$$

$$t_f = -\frac{h_0}{u_0} + 2t_s$$