Probability Cheat Sheet

Two Schools of Thought

Frequentist: based on repeatable, random experiments In this view the probability of an event is defined as the limiting proportion of times the event would occur given many repetitions. For example, defining the probability of a coin landing on heads by measuring the proportion of times a fair coin lands on its head out of the total times it is tossed. Limited to scenarios where frequent repetitions of the same random experiment are possible. Cannot be applied to novel scenarios with no clear sampling frame for repetitions.

Bayesian: allows for incorporation of investigator's reasoning. Instead of exclusively relying on knowledge of the proportion of times an event occurs in repeated sampling, this approach allows the incorporation of subjective knowledge, such as historical information from similar experiments as the one under study, information from experiments related to the one under study, an educated guess about outcomes, or even, subjective beliefs of the investigator related to the problem under study. These so-called prior probabilities are then updated in a rational way after data are collected.

Essentials of Probability Theory

random trial: process or experiment that has two or more possible outcomes whose occurrence cannot be predicted

sample space: list of all possible outcomes of a random trial

event: any potential subset of the sample space

the probability of an event is the proportion of times the event would occur if we repeated a random trial over and over again under the same conditions

Fundamental Rules of Probability

Probability is always positive: the values of a probability must range from 0 to 1

For a given sample space, the sum of all of the probabilities is always 1: This also allows for the calculation of the complement of an event. P(not A)=1-P(A)

For mutually exclusive events, P(A or B)=P(A)+P(B).

Counting

Many times in probability you will need to be able to quantify things before assigning probabilities and applying counting methods is essential in doing this. The mathematical theory of counting is called combinatorial analysis.

Fundamental Principle of Counting (Multiplication Principle): if the number of outcomes of experiment 1 is m, and then number of outcomes of experiment 2 is n, then the total number of outcomes for the two experiments is m * n.

Permutations (Order is Important): A permutation of objects occurs when objects are arranged so that order is important. Mathematically the formula for a permutation or an arrangement of r out of n distinct objects (order is important) is:
$P_{n,r}$=n!/(n-r)!

Combinations (Order is Unimportant): A combination of objects occurs when objects are selected and order of arrangements is not important. Mathematically the formula for a combination of selecting r out of n distinct objects (order unimportant) is:
Cn,r=n!/r!(n-r)!

<u>Useful Probability Bits:</u>

Independence: two events are independent if: (1) the occurrence of one does not change the probability that the second will occur (2)the probability of one event depends on the result of another event

And versus Or: if you would use *or* in the sentence, add
P(A or B)=P(A)+P(B)
(if A and B are mutually exclusive)
if you would use *and* in the sentence, multiply
P(A and B)=P(A)xP(B)
(if A and B are independent)

Law of Total Probability: law of total probability: if we want to know the overall probability of an event, we sum its probability across every possible condition, weighted by the probability of that condition
P(X)=ΣP(Y)P(X|Y)

General Probability Rule: probability that both of two events occur, even if the two are dependent
P(A and B)=P(A)/P(A|B)

Bayes' Theorem: powerful mathematical relationship about conditional probability
P(A|B)=P(B|A)P(A)/P(B)